

14159265358979323846264338327950288419716939937510582097494459230781640628620899862803482534211706798214808651328230664709384460950582
 2317253594081284811174502841027019385211055596446229489549303819644288109756659334461284756482337867831652712019091456485669234603486104

Fun with π A Potpourri of Pilish Algorithms

Michael Keith
March 2025

Not A Wake
A Great Embedding of π 's Digits Into
words For 10000 Decimals

Not A Muse
A Movie Involving π 's Digits

5432 6648
 2133 9360
 7260 2491
 4127 3724
 5870 0660
 6315 5881
 7488 1520
 9209 6282
 9254 0917
 1536 4367
 3925 9036
 1011 3305
 3054 8820
 4665 2138
 4146 9519
 4151 1609
 4330 5727
 0365 7595
 9195 3092
 1861 1738
 1932 6117
 9310 5118
 5480 7446
 2379 9627
 4956 7351
 3857 5272
 4891 2279
 3818 3011

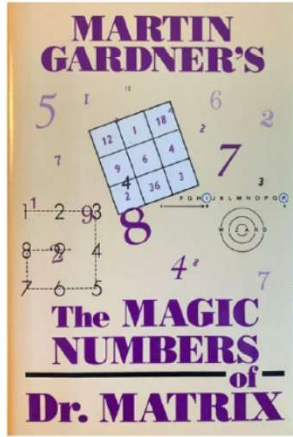
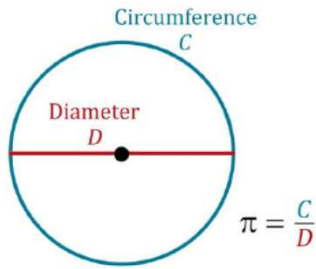
0491298336733624406566430860213949463952247371907021798609437027705392171762931767523846748184676694051320005681271452635608277857713427
 577896091736371787214684409012249534301465495853710507922796892589235420199561121290219608640344181598136297747713099605187072113499999

Fun with Pi: A Potourri of Pilish Algorithms

Presented at Princeton Chapter of ACM – March 20, 2025.

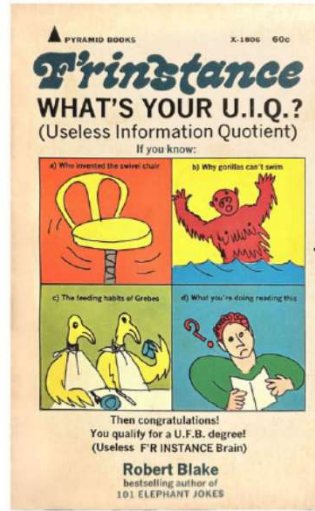
3141592653589793238462643383279502884197169399375105820974944592307816406286208998628034825342117067982148086513282306647093844609550582

Confessions of a Pi Addict



Correctly interpreted, you know, π conveys the entire history of the human race.

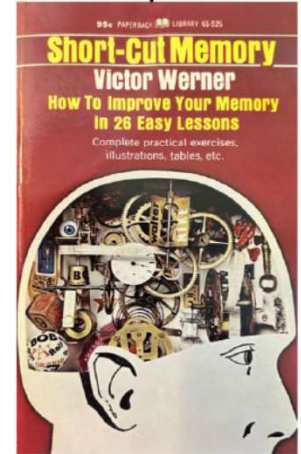
- Dr. Matrix, p. 61



For all Math Lovers — Here is π (pi) to a thousand decimal places:

3.141592653589793238462643383279502884197
16999375105820974944592307816406286208
996628034825342117067982148086513282306
6470938446095505822317253940812848117
45028410270193852110559464622948594930
38196442881097566593346128475648233785
78316527120190914564856923460348610454
326648213393607260249141273724587006606
31558817488152092062829254091715364367
89259360011330530548820466521384146951
941511609433057270365759591953092186117
381952611793105118548074462379962749567
351885752724891227938183011949129833673
36244055643086021394946395224737197002
1798609437027705392171769317652384674
818467669405132000568127145263560627785
771342757789609173637178721468440901224
95343014654958537105072279689258923542
019956112129021960864034418159813629774
77130996051870211349999998372978049951
059731732816096318595024459455346908302
642522308253344685033261931188171010003
13783875288638733208381420617177669147
30359825349042875545873115956386382383
78755375195778185780532171226806513001
9227866111959092164201989

The Major System



9491298336733624406566430860213949463952247371907021798609437027705392171762931767523846748184676694051320005681271452635608277857713427
5778960917363717872146844090122495343014654958537105079227968925892354201995611212902196086403441815981362977477130996051870721134999999

Slide 2: This talk is somewhat personal as well as technical. These three books from my childhood were directly responsible for starting my life-long interest in the number π . Martin Gardner's columns in *Scientific American* were an inspiration in general, but especially the ones featuring his fictional "Dr. Matrix", which tended to be extra fun and sometimes involved π . In the middle is a book of humorous trivia that happened to be the first place I saw many digits of π (there are 1001 there). The book on the right piqued my interest in memorizing the digits of π – which we'll talk more about later.

3141592653589793238462643383279502884197169399375105820974944592307816406286208998628034825342117067982148086513282306647093844609550582	6648
2317253594081284811174502841027019385211055596446229489549303819644288109756659334461284756482337867831652712019091456485669234603486104	9360
5432	2491
2133	3724
7260	0660
4127	5881
5870	1520
5315	6282
7488	0917
9209	4367
9254	9036
1536	3305
8925	8820
0011	2138
3054	9519
4665	1609
4146	5727
4151	7595
4330	3092
0365	1738
9195	6117
1861	5118
1932	7446
9310	9627
5480	7351
2379	5272
4956	2279
8857	3011
4891	
3818	
0491298336733624406566430860213949463952247371907021798609437027705392171762931767523846748184676694051320005681271452635608277857713427	
5778960917363717872146844090122495343014654958537105079227968925892354201995611212902196086403441815981362977477130996051870721134999999	

The First 31 digits

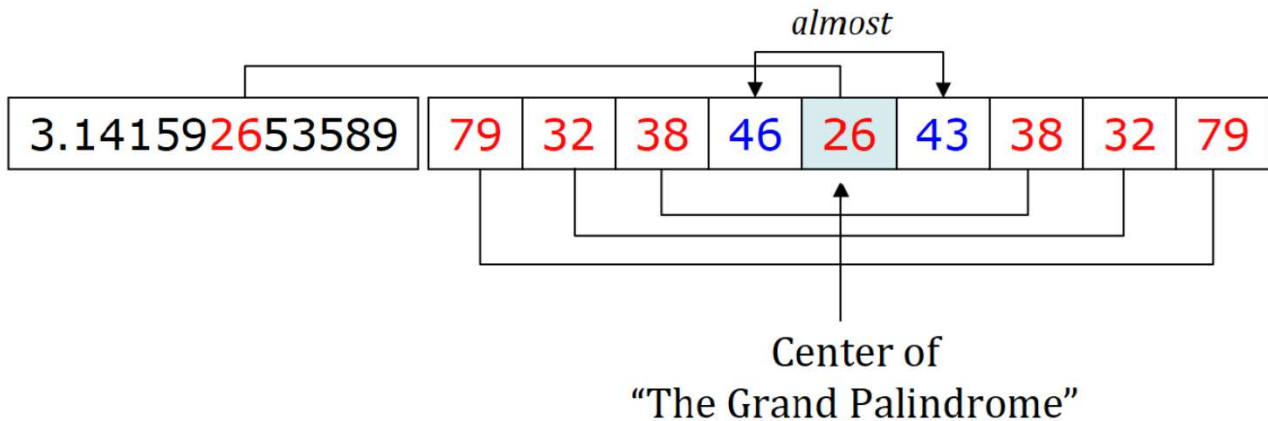
Observation #1: Look at all those **ABA**'s!

3.**141**59 26 **535** 8 **979** **323** 84 **626** 43 **383** 279...

Slide 3: Here (and on the next slide) are two interesting features of the first 31 digits. The probability of an ABA pattern (in random digits) is 1/10, so in 30 digits we'd expect to see 3 ABAs on average, but there are 6 of them. It's a catchy pattern to say so it helps in memorizing these initial digits.

The First 31 digits

Observation #2: from Martin Gardner's "Dr. Matrix":



Slide 4: Another way to look at these was pointed out by M. Gardner. After the 3.141592653589 is a series of 18 digits which, in pairs, is nearly a palindrome. It's exact except for 46/43, which are almost a match. Note that the 26 in the middle of the palindrome harkens back to the earlier 26.

The First 100 Digits

3.141592653589 79 32 38 46 26 43 38 32 79 5028 84 1971

permutations of 5280
(feet in a mile)

“comma”

years

693 993 7510 5820 974 944 5923078164 0628 6208 998 6280

rhyme (2-syl) rhyme rhyme

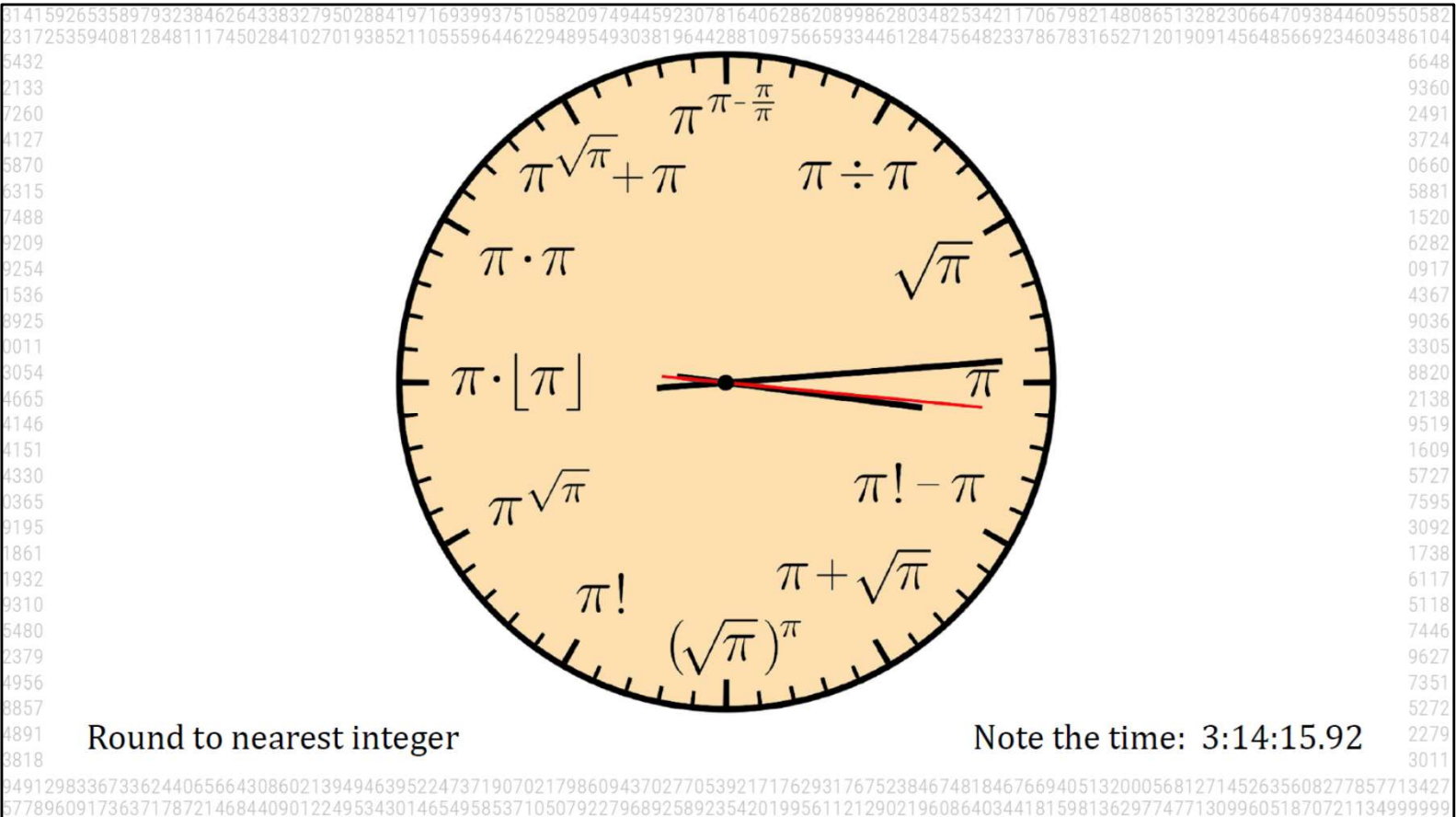
348 253 4211 7067

Fibonacci descending powers of 2

permutation of 0123456789
(expectation = once every $10^{10}/10! = 2756$ digits)

$2\pi = 6.28$
with a “sliding 0”

Slide 5: Even the first 100 digits have enough interesting patterns to be fairly easy to memorize. The annotations show some of allusions I use to help remember them.



Slide 7: Speaking of clocks, here's one where the 1-to-12 hour numbers are represented using only the number π . (You have to implicitly round each to the nearest integer.) Another nice observation is that at “ π time” on a clock (that is, 3:14:15.92) all three hands fit in a very small angle.

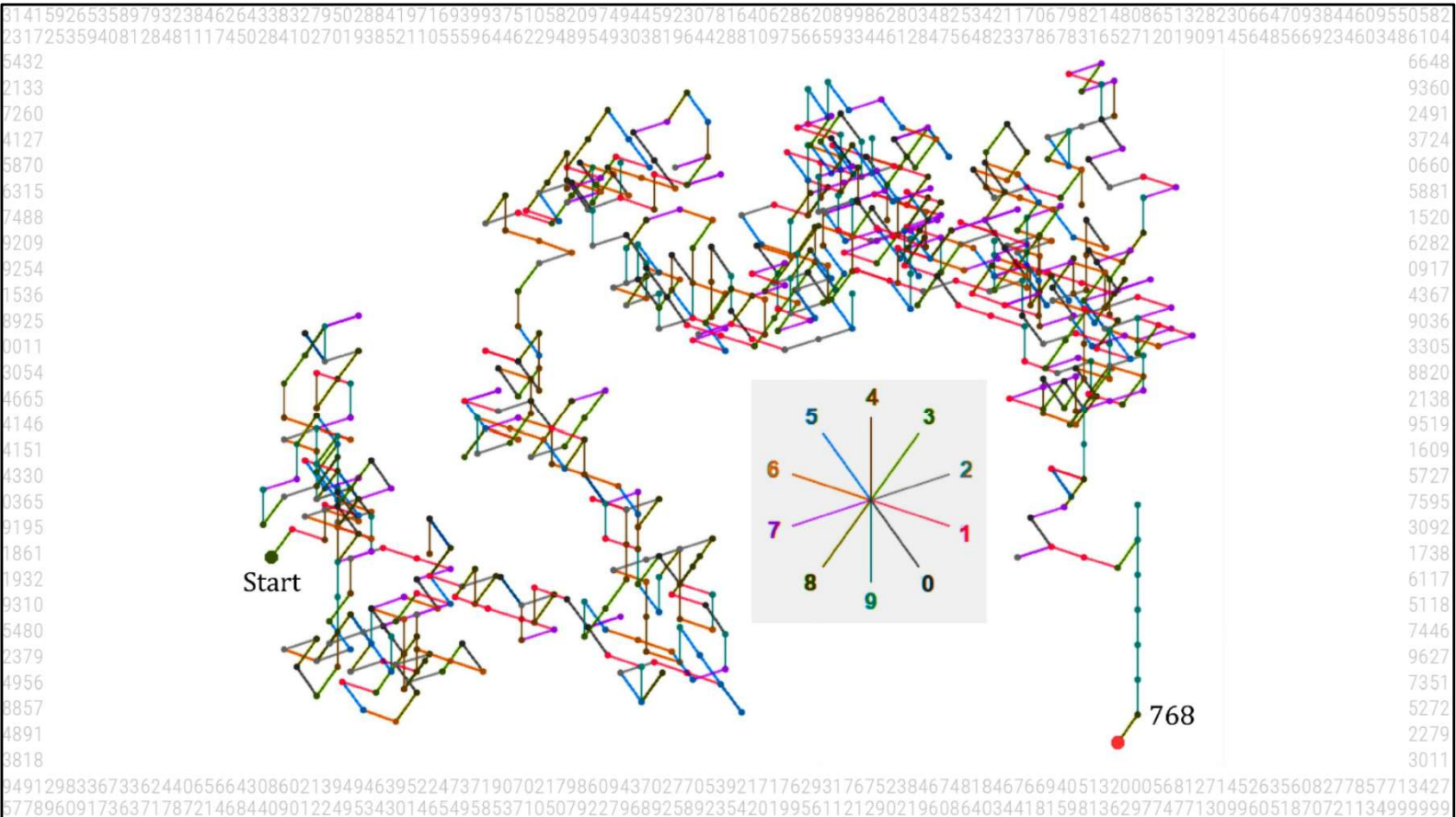
3141592653589793238462643383279502884197169399375105820974944592307816406286208998628034825342117067982148086513282306647093844609550582			
2317253594081284811174502841027019385211055596446229489549303819644288109756659334461284756482337867831652712019091456485669234603486104			
5432			6648
2133			9360
7260			2491
4127			3724
5870			0660
6315			5881
7488			1520
9209			6282
9254			0917
1536			4367
8925			9036
0011			3305
3054			8820
4665			2138
4146			9519
4151			1609
4330			5727
0365			7595
9195			3092
1861			1738
1932			6117
9310			5118
5480			7446
2379			9627
4956			7351
8857			5272
4891			2279
3818			3011
0491298336733624406566430860213949463952247371907021798609437027705392171762931767523846748184676694051320005681271452635608277857713427			
5778960917363717872146844090122495343014654958537105079227968925892354201995611212902196086403441815981362977477130996051870721134999999			

The First 768 Digits

$$768 = 2^8 \times 3$$

314159
 2653589793238462
 6433832795028841971693
 99375105820974944592307816
 4062862089986280348253421170
 67982148086513282306647093844609
 5505822317253594081284811174502841
 027019385211055964462294895493038
 196442881097566593344612847564823378
 678316527120190914564856692346034861
 04543266482133936072602491412737245870
 06606315588174881520920962829254091715
 36436789259036001133053054882046652138
 41469519415116094330572703657595919530
 92186117381932611793105118548074462379
 96274956735188575272489122793818301194
 912983367336244065664308602139494639
 522473719070217986094370277053921717
 6293176752384674818467669405132000
 5681271452635608277857713427577896
 09173637178721468440901224953430
 1465495853710507922796892589
 23542019956112129021960864
 0344181598136297747713
 0996051870721134
 999999

Slide 8: One of the most well-known and remarkable things in π 's digits is the 999999 that appears at digits 763 to 768 (see bottom line of the disc of digits). There are three "runs" of three equal digits in this range (shown in red), but no longer runs other than the 999999. This is clearly surprising, but how much?



Slide 9: Another “graphic design” sidebar. We make a “walk” diagram (like a random walk, but using the digits of π to drive it) with 10 different walk directions, one for each digit value. Of course the 999999 is very obvious at the lower right (I’ve also included the 8 that’s just after the 999999).

Probability of a run: an exact formula

Let $\rho(n,r,b)$ = probability that a “run” of equal digits (of length r) starts within the first n digits after the radix point of a random base- b real number. Then:

$$\rho(n,r,b) = P(n+r-2, r-1, \frac{1}{b})$$

where $P(n,r,p) = 1 - A(n,r,p) + p^r A(n-r,r,p)$

$$\text{and } A(n,r,p) = \sum_{k=0}^{\lfloor n/(r+1) \rfloor} (-1)^k \binom{n-kr}{k} ((1-p)p^r)^k$$

$P(n,r,p)$ is the probability of a run of r successes in n trials, where the probability of success on each trial is p .

[see Villarino, *The probability of a run*, Math. Gazette, Mar. 2007]

Slide 10: Let's do a deep dive into the 999999. We'd like to know how unlikely it really is for this to appear in the first 768 digits, and would also like to know: is there another run of equal digits later in π that's even more remarkable? To quantify this, note that π 's digits look like statistically independent uniform random digits. So to answer “how remarkable is some pattern in π ?” we can ask “what's the probability of that pattern occurring in a uniformly random string of digits?”. One can answer this roughly using approximations, but amazingly there's an exact formula, shown here.

Other runs vs. the 999999

- Denote by N the benchmark probability of the 999999, a run of length 6 starting within the first 762 digits after the decimal point:

$$N = \rho(762, 6, 10) = 0.0068358667... \approx \mathbf{1/146}$$

- The relative rarity of some other (n, r, b) run is

$$R = \frac{N}{\rho(n, r, b)}.$$

- A run is more remarkable than the 999999 if

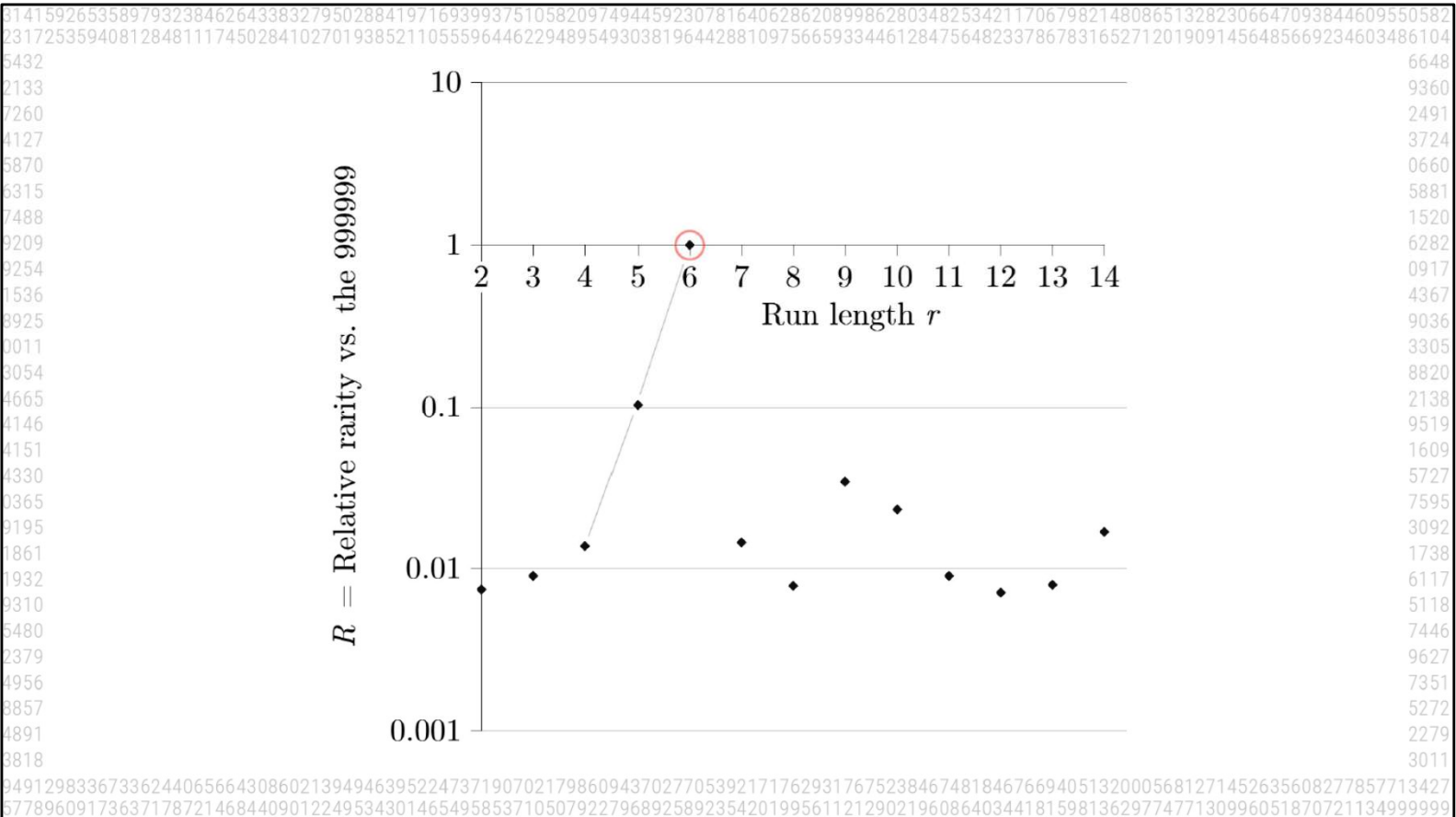
$$R > 1$$

Slide 11: Using that formula we find that the probability of six identical digits in a row (doesn't matter what the actual digit is) appearing somewhere in the first 768 digits is $N \approx 1/146$, which matches our intuition – it's quite unlikely. Define R as the relative probability of some other run compared to N . We seek another run with $R > 1$.

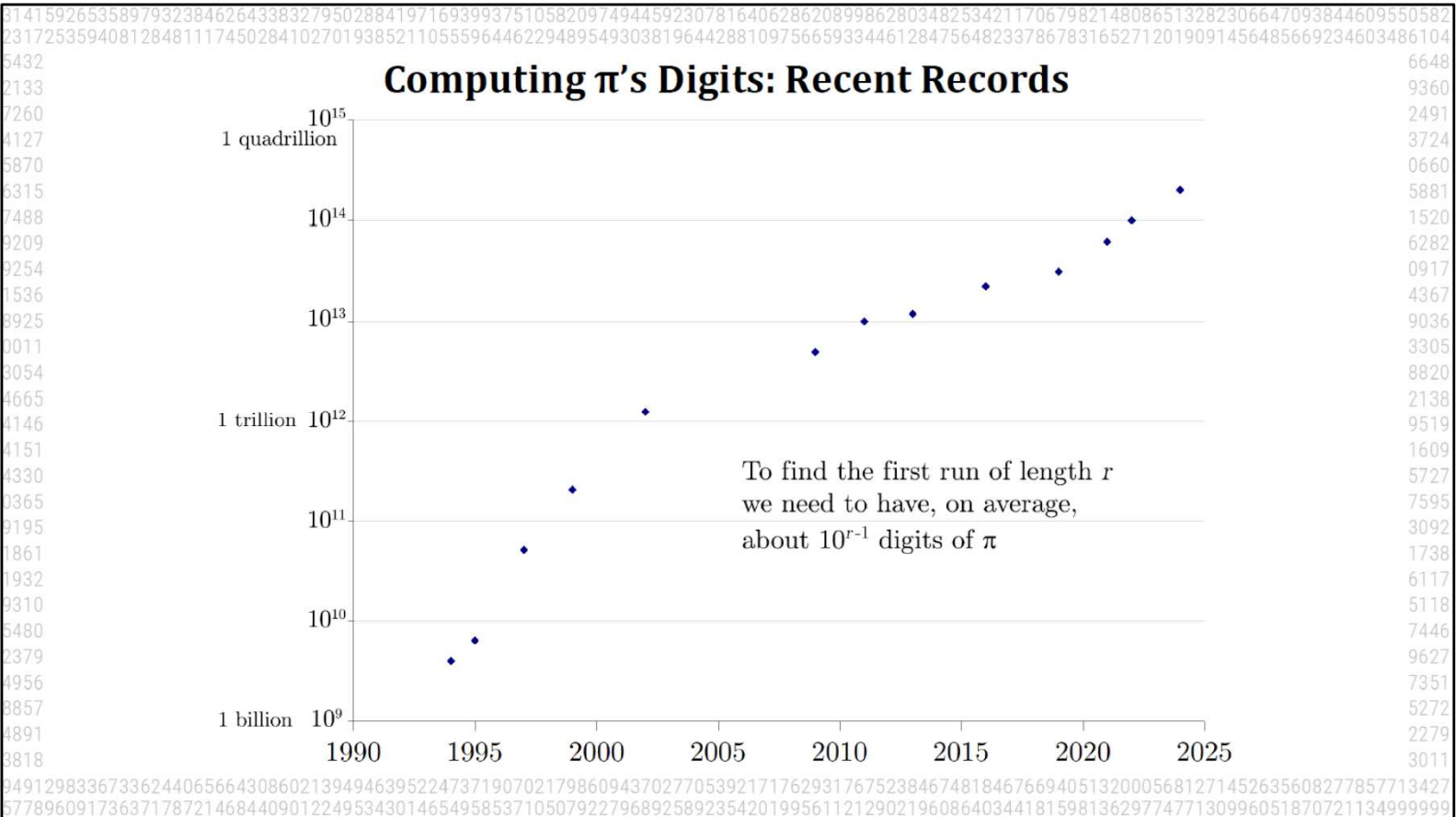
Earliest run of r consecutive equal digits in π

r	Starting position	
2	25	314159265358979323846264 33 832795028841971693993751
3	154	955058223172535940812848 111 74502841027019385211055
4	763	9774771309960518707211349999 9983729780499510597317
5	763	9774771309960518707211349999 9983729780499510597317
6	763	9774771309960518707211349999 9983729780499510597317
7	710, 101	879578929481590006873537 333333 8638139084209948725
8	22, 931, 746	466191197618918591274369 44444444 364031136185917604
9	24, 658, 602	963899194082457597185304 77777777 24846769425931046
10	386, 980, 413	389979222684034599643705 66666666 9143667924631302
11	15, 647, 738, 229	138297941926915829852845 1111111111 489929932710771
12	368, 299, 898, 267	753824828890356168372651 7777777777 62660543965828
13	2, 164, 164, 669, 333	890030662602837484934396 888888888888 4371300344594
14	5, 758, 910, 552, 710	584798672924370534391064 999999999999 167658104623

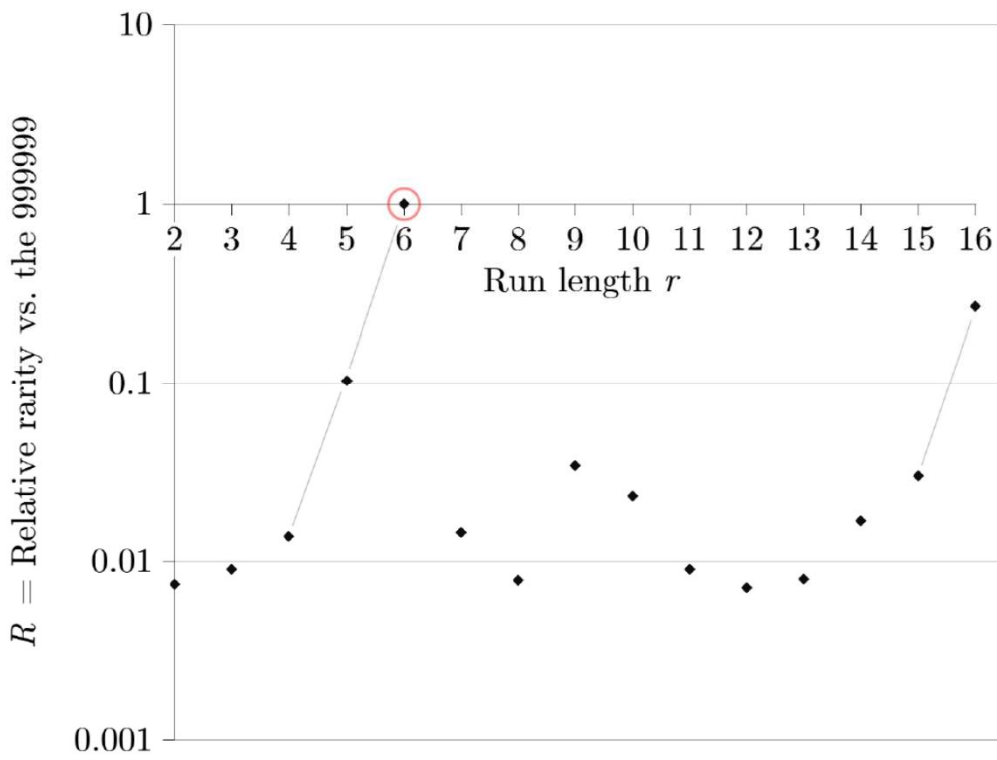
Slide 12: Here are the first runs of length r from $r = 2$ to 14. Note that the 999999 is simultaneously the first run of length 4, 5, and 6, and this kind of thing does not occur again up to $r = 14$.



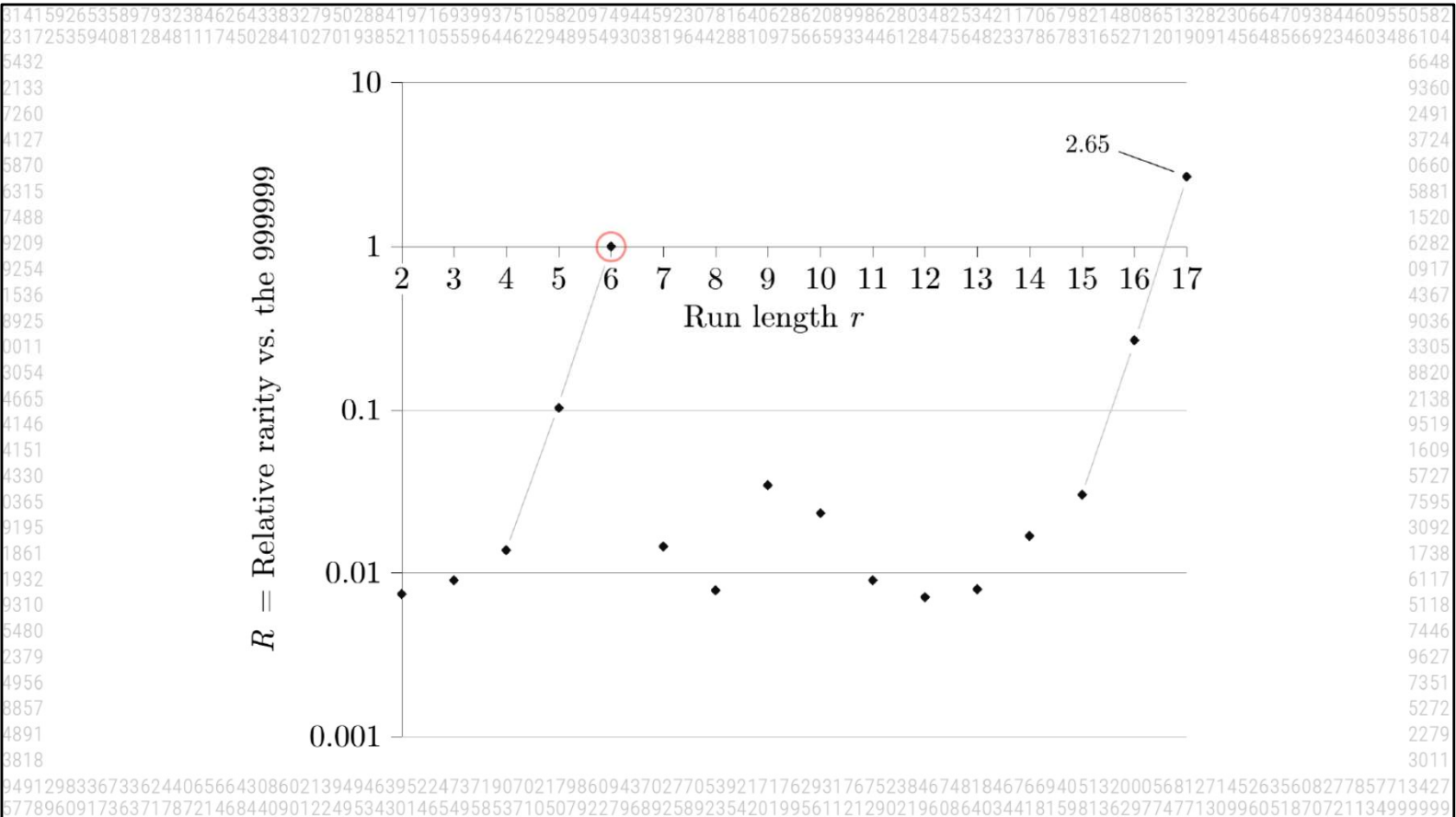
Slide 13: Here are the actual values of R for those runs (note that vertical scale is logarithmic). As you can see, there aren't any with $R > 1$.



Slide 14: Sidebar: how many digits of π do we currently know? This shows the answer – the current record computation is 202 trillion digits, from June 2024. We can see from this that the graph on the previous page is getting close to the point (though not quite there) when we'll run out of known digits.



Slide 15: BUT...guess what, we do know a few more data points – here we’ve added the next two. Note that they start another set of “embedded” runs like the $r = 4,5,6$ set. If only there was one more in the set we would get one with $R > 1$...



Slide 16: Yes! Unbelievably, there is one more in the set, with the first runs of length 15, 16, and 17 all occurring at the same place, and the $r = 17$ run has $R > 1$! Specifically, it's $R = 2.65$. Note that $r = 17$ is the last one currently known – humans haven't computed enough digits of π to find any first run with $r > 17$.

r	Starting position	
2	25	314159265358979323846264 33 832795028841971693993751
3	154	955058223172535940812848 111 74502841027019385211055
4	763	97747713099605187072113499999983729780499510597317
5	763	97747713099605187072113499999983729780499510597317
6	763	97747713099605187072113499999983729780499510597317
7	710, 101	879578929481590006873537 333333 8638139084209948725
8	22, 931, 746	466191197618918591274369 44444444 364031136185917604
9	24, 658, 602	963899194082457597185304 777777777 24846769425931046
10	386, 980, 413	389979222684034599643705 6666666666 9143667924631302
11	15, 647, 738, 229	138297941926915829852845 1111111111 1489929932710771
12	368, 299, 898, 267	753824828890356168372651 7777777777 62660543965828
13	2, 164, 164, 669, 333	890030662602837484934396 888888888888 4371300344594
14	5, 758, 910, 552, 710	584798672924370534391064 9999999999999 167658104623
15	28, 642, 224, 609, 577	528214330659792827152450666666666666666666947789408
16	28, 642, 224, 609, 577	528214330659792827152450666666666666666666947789408
17	28, 642, 224, 609, 577	528214330659792827152450666666666666666666947789408

Slide 17: Here are the actual digits for all the known first runs.

What About Other Bases?

- Like any number, π can be written in other bases (not just base 10):

- Base 10 uses digits 0 to 9 - Base b uses digits 0 to $b-1$

Base 5:	3 . 0 3 2 3 2 2 1 4 3 0 3 3 4 3 2 4 1 1 2 4
Base 6:	3 . 0 5 0 3 3 0 0 5 1 4 1 5 1 2 4 1 0 5 2 3
Base 7:	3 . 0 6 6 3 6 5 1 4 3 2 0 3 6 1 3 4 1 1 0 2
Base 8:	3 . 1 1 0 3 7 5 5 2 4 2 1 0 2 6 4 3 0 2 1 5
Base 9:	3 . 1 2 4 1 8 8 1 2 4 0 7 4 4 2 7 8 8 6 4 5
Base 10:	3 . 1 4 1 5 9 2 6 5 3 5 8 9 7 9 3 2 3 8 4 6
Base 11:	3 . 1 6 1 5 0 7 0 2 8 6 5 10 4 8 5 2 3 5 2 1
Base 12:	3 . 1 8 4 8 0 9 4 9 3 11 9 1 8 6 6 4 5 7 3 10
Base 16:	3 . 2 4 3 15 6 10 8 8 8 5 10 3 0 8 13 3 1 3 1 9
Base 28:	3 . 3 27 0 6 21 18 24 19 25 21 27 10 7 7 4 7 11 12 24 13
Base 71:	3 . 10 3 54 40 24 34 69 42 54 4 28 24 15 2 0 63 45 45 32 53

...

The **888** in base 16 has $R = 0.265... \approx \frac{1}{4}$

Slide 18: So we've completely explored base 10 for runs of equal digits, but what about other bases? Again remarkably, there's a fairly nice run in hexadecimal, since π in base 16 is 3.243F6A888..., which has the run 888 very early, at the 8th digit. This doesn't have $R > 1$, but its R value is about $1/4$, which is quite respectable (only 4 times less remarkable than the 999999 in base 10).

314159265
231725359
5432
2133
7260
4127
5870
6315
7488
9209
9254
1536
8925
0011
3054
4665
4146
4151
4330
0365
9195
1861
1932
9310
5480
2379
4956
8857
4891
3818
049129833
577896091

609550582
603486104
6648
9360
2491
3724
0660
5881
1520
6282
0917
4367
9036
3305
8820
2138
9519
1609
5727
7595
3092
1738
6117
5118
7446
9627
7351
5272
2279
3011
857713427
134999999

3.	24	85	125	119	51	8	119	153	80	156	102	84	147	101	1	31
	108	134	8	45	54	19	142	80	134	1	75	52	162	161	113	49
	103	88	68	93	154	61	5	110	125	27	147	123	152	6	94	19
	135	97	151	77	37	71	17	60	121	60	17	33	9	36	33	100
	92	36	41	63	77	35	64	163	152	78	172	8	97	99	94	71
	55	23	48	154	19	53	87	151	27	99	52	42	145	68	9	36
	53	21	152	148	14	101	31	51	139	145	56	148	73	20	46	136
	100	147	89	127	167	163	85	125	23	7	15	101	50	50	50	50

π in
Base 173

 $R \approx 286$

$n = 125$



Slide 19: Now we're talking! This run of four 50's at the 125th digit in base 173 has an R value of 286. Just for fun, I assigned each of the base-173 digits (0, 1, ... 172) to a different country, giving the representation of π in base 173 shown at the bottom using country flags.

Base 209259756

$$\pi = 3.29629644 \ 29629644 \ 186455662 \ 92370170 \ 151764814$$

$$R = 1,430,472$$

Base

1727491725250647435729787945728673850319534083434052744734024329932445544365076469341423533662286038148

$\pi =$

$$3.244600137432649199445625995435916446443754804354586147235120057381283953469046519861204385023377698504 \\ 244600137432649199445625995435916446443754804354586147235120057381283953469046519861204385023377698504 \\ 918261859806935457005061340389877681555177462528812198083274952539900166673450269415216810894908653353$$

$$R \approx 1.18 \times 10^{100} \leftarrow \text{just over 1 googol!}$$

Conjecture: 3.xx examples exist with arbitrarily large R.

Slide 20: You think R=286 is good? – you ain’t seen nothing. These two examples have a run (of length 2) immediately after the decimal point, which is the place where the R value is highest. The top one has R greater than 1 million while the second one has R greater than 1 googol! It is conjectured that an infinite number of examples like this exist, so there are examples with arbitrarily large R.

A Beautiful Formula

David and Gregory Chudnovsky (+ Ramanujan), 1988:

$$\pi = \left(12 \cdot \sum_{n=0}^{\infty} \frac{(-1)^n (6n)!}{(3n)!(n!)^3} \cdot \frac{13591409 + 545140134n}{640320^{3n+3/2}} \right)^{-1}$$

- Each term in the sum adds 14+ decimal digits
- For full speed: (1) Do the summation with “binary splitting”
(2) Use fast integer multiplication (Schönhage-Strassen)
- With these, the complexity is $O(n \cdot (\log n)^3 \cdot \log \log n)$, where $n = \#$ of digits
- Has been used for almost all world-record π computations since ~1990

Slide 21: Change of topic: beautiful formulas or algorithms or approximations to π . The equation shown here is the basis of the Chudnovsky Algorithm for computing the digits of π , which has been state-of-the-art since around 1990 (as no better algorithm has been found since then). It’s interesting to note that the speed of the algorithm is now not the bottleneck when computing π – instead, the bottleneck is the speed and capacity of the SSDs used for mass storage during the computation.

A Beautiful Algorithm (in C language)

The Wagon/Rabinowitz Spigot (with the "predigit hack")

```

int main() // Hard-coded for 768 digits
{
    int64_t accum = 0, prev = 0, length = 2592, stride = 27;
    int64_t a[2592], k, denom, digit, base = 1e8;

    for (k=1; k<=length; k++) a[k] = base/5;

    while (length > 0) {
        for (k = length-1; k >= 1; k--) {
            accum = accum * k + (a[k] * base);
            denom = 2*k - 1;
            a[k] = accum % denom;
            accum = accum / denom;
        }
        digit = prev + accum/base;
        prev = accum % base; accum = prev;
        length -= stride;
        printf("%08ld", digit);
    }
}

```

Complexity is $O(n^2)$
Can compute 100K digits in ~15 seconds

Slide 22: This is a nice, and especially fun, algorithm for computing the digits that is very short and very simple. Note that it only requires one 64-bit integer array and seven 64-bit integer variables, and only 10 or so lines of code. For more info online, search for “spigot algorithm for π ”.

How it works

$$\pi = 3 + \frac{1}{10} \left(1 + \frac{1}{10} \left(4 + \frac{1}{10} \left(1 + \dots \right) \right) \right)$$

$$\pi = \sum_{n=0}^{\infty} \frac{2 \cdot n!}{(2n+1)!!}$$

$$\Rightarrow \pi = 2 + \frac{1}{3} \left(2 + \frac{2}{5} \left(2 + \frac{3}{7} \left(2 + \dots \right) \right) \right)$$

Slide 23: This is a brief summary of how it works. At the top is an expression of π as a nested summation showing that the digits (3 1 4 1...) are a representation of π in base 10 (because of the 1/10s). Below that, a known summation for pi can be transformed into an expression that's a mixed-radix representation of π , with bases 1/3, 2/5, 3/7, etc. Note that in this base π is simply 2.2222...! This explains why the code can be so simple – it's merely converting π from 2.222... in the mixed-radix base to base 10.

Some "code golfing" gives this tiny C version

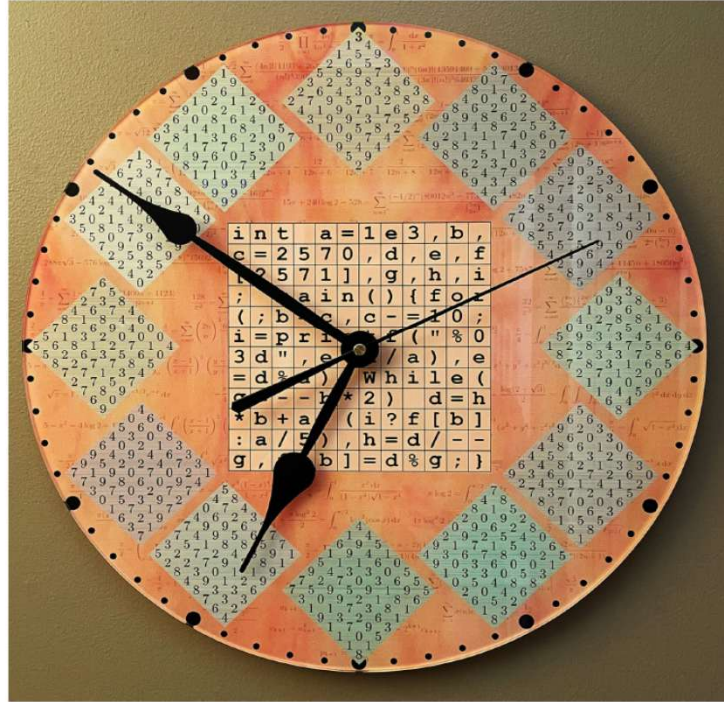
(by Dik T. Winter, Achim Flammenkamp, Christoph Haenel, and others)

$$(12 \times 11) - 1 = 131$$

a	[2	7	0	2]	,	b	,	c	=
2	7	0	2	,	d	,	f	=	1	e	4
,	e	,	g	,	h	;	m	a	i	n	(
)	{	f	o	r	(;	b	=	c	-	=
1	4	;	h	=	p	r	i	n	t	f	(
"	%	0	4	d	"		,	e	+	d	/
f))	f	o	r	(e	=	d	%	=
f	;	g	=	-	-	b	*	2	;	d	/
=	g)	d	=	d	*	b	+	f	*	(
h	?	a	[b]	:	f	/	5)	,
a	[b]	=	d	%	-	-	g	;	}

Slide 24: "Code golf" is the game of trying to reduce an algorithm's source code to the smallest possible number of characters. In the C language, the code on slide 22 can be squeezed into 131 characters, which I have arranged here in a 12 x 11 rectangle with the gray square near the center not used.

A different version with $(12 \times 12) - 3 = 141$ characters



Slide 25: I used a different version of this code to make this actual physical clock. The 12 large diamonds serving as hour markers on the clock contain the program's output (768 digits = 12 sets of 64 digits).

3141592653589793238462643383279502884197169399375105820974944592307816406286208998628034825342117067982148086513282306647093844609550582			
2317253594081284811174502841027019385211055596446229489549303819644288109756659334461284756482337867831652712019091456485669234603486104			
5432			6648
2133			9360
7260			2491
4127			3724
5870	\documentclass[11pt]{article}		0660
6315	\usepackage[letterpaper]{geometry}		5881
7488	\usepackage{lmodern, luacode}		1520
9209	\begin{document} \Large \centering		6282
9254	Calculating π using a \LaTeX		0917
1536	document (namely, this one!)		4367
8925	\par \bigskip \large \begin{luacode}		9036
0011	Q,o,oo,OO,0,Qo={},2688,0,0,0,1 for	3141592653589793238462643383279502884197169399375105820974944592	3305
3054	Oo=0,o do Q[Oo]=2000 end for oO=o	3078164062862089986280348253421170679821480865132823066470938446	8820
4665	,14,-14 do O=0 Oo=oO while(1)do	0955058223172535940812848111745028410270193852110555964462294895	2138
4146	O=O+Q[Oo]*10000 oo=2*Oo-1 Q[Oo]=	4930381964428810975665933446128475648233786783165271201909145648	9519
4151	O%oo O=O//oo Oo=Oo-1 if(Oo==0)then	5669234603486104543266482133936072602491412737245870066063155881	1609
4330	break end O=0*Oo end tex.sprint(7488152092096282925409171536436789259036001133053054882046652138	5727
0365	string.format("%04d",OO+O//10000))	4146951941511609433057270365759591953092186117381932611793105118	7595
9195	if Qo%16<1 then tex.print(" ")	5480744623799627495673518857527248912279381830119491298336733624	3092
1861	end Qo=Qo+1 OO=O%10000 end	4065664308602139494639522473719070217986094370277053921717629317	1738
1932	\end{luacode} \end{document}	6752384674818467669405132000568127145263560827785771342757789609	6117
9310		1736371787214684409012249534301465495853710507922796892589235420	5118
5480		1995611212902196086403441815981362977477130996051870721134999999	7446
2379			9627
4956			7351
8857			5272
4891			2279
3818			3011
9491298336733624406566430860213949463952247371907021798609437027705392171762931767523846748184676694051320005681271452635608277857713427			
5778960917363717872146844090122495343014654958537105079227968925892354201995611212902196086403441815981362977477130996051870721134999999			

Heavy-meta π in LuaLaTeX

Calculating π using a \LaTeX document (namely, this one!)

Slide 26: Another fun use for a short program like this is to compute π with the source code of a LaTeX document. The tiny code on the left (which is complete, valid LuaLaTeX source) produces the document at the right, in which the first 768 digits are computed by the process of compiling the document source.

Some Beautiful Approximations

- Let efficiency $E = (\# \text{ of correct } \pi \text{ digits}) / (\# \text{ of digits in the expression})$

$$\frac{355}{113} = 3.141592920\dots \quad E = 7/6 \approx 1.2$$

$$\sqrt{\sqrt{97 + \frac{9}{22}}} = 3.14159265258264\dots \quad E = 10/5 \approx 2.0$$

$$\frac{\ln(640320^3 + 4! + 6!)}{\sqrt{163}} = 3.141592653589793238462643383279726\dots \quad E = 31/12 \approx 2.6$$

$$\pi \approx \ln\left(\frac{[(11 + \sqrt{106} + \sqrt{223 + 22\sqrt{106}})(3 + \sqrt{8})(7 + \sqrt{53})]^{12}}{4^9} + 4!\right) \div \sqrt{762} \quad E = 73/26 \approx 2.8$$

$$= 3.141592653589793238462643383279502884197169399375105820974944592307816406333\dots$$

Slide 27: Yet another fun recreation: find expressions containing numbers and math symbols that approximate the value of π , then “score” them by an efficiency metric that compares the number of correct digits in the approximation to the number of digits in the expression. Here are four examples with the desired feature of having $E > 1$. Every example but the first is based on the work of the Indian mathematician Srinivasa Ramanujan.

Pilish

- English with an added constraint:

The number of letters in each word follows the digits of π .

“How I need a drink (alcoholic, of course)

3 1 4 1 5 9 2 3

after the heavy lectures involving quantum computing.”

5 8 9 7 9 7 9

- Very much in the spirit of Oulipo, the (mostly) French literary group

Slide 28: New topic: the wonders of Pilish. As shown here, Pilish is English but with each word length following the sequence of π 's digits. This example is an old classic that I've updated to be very modern by changing the word “mechanics” to “computing”.

Piem #1

But a time I spent wandering in gloomy night,
Yon tower, tinkling chimewise, loftily opportune;
Out, up, and together came twelve to Sunday rite,
And one silently off to observe plenilune.

*Joseph Shipley (1960) &
Michael Keith (later)*

(31 digits)



Slide 29: Pilish can even be wrangled to handle additional constraints, such as here where we have a four-line poem with ABAB rhyme scheme. This is a joint composition by me and American writer Joseph Shipley. BTW, “plenilune” means “full moon”.

Piem #2

See a moon – a globe brilliant in flight –
Cross the great Jupiter’s celestial ellipse;
Computers use pi – one constant they recite –
To create data now for tracking sun at eclipse.

*Howard Bergerson (1965) &
Michael Keith (later)*

(30 digits)



Slide 30: Another example, and another collaboration – this time with famous wordplay expert Dmitri Borgmann. Surprisingly, this one also has an astronomy theme. And it self-referentially includes “pi”.

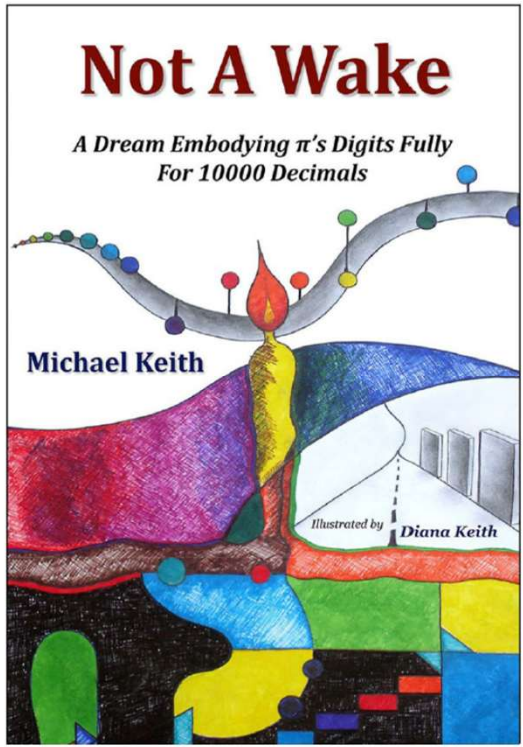
Dealing with 0's - and one more handy rule

31415926535	Now I fall, a tired suburban in liquid under the trees
8979323846	Drifting alongside forests simmering red in the twilight over Europe.
2643383279	So scream with the old mischief, ask me another conundrum
502884197	About bitterness of possible fortunes near a landscape Italian.
169399375	A little happiness may sometimes intervene but usually fades.
105820974	A missionary cries, striving to understand worthless, tedious life.
9445923078	Monotony's lost amid ocean movements as the bewildered sailors hesitate.
16406286208	I become salt, submerging people in dazzling oceans of enshrouded unbelief.
998	Christmas ornaments conspire.
628034	Beauty is, somewhat inevitably now, both
82534211	Feelings of faith and eyes of rationalism .

Slide 31: To be complete we must mention a few more rules:

- (a) A 10-letter word represents the digit 0,
- (b) Words of more than 10 letters are allowed; such a word stands for two consecutive digits (e.g., an 11-letter word represents (1, 1), and 12-letter word is (1, 2), etc.

Here the 10- and 11-letter words are in red.



Now I fall, a tired suburban in liquid under the trees
 Drifting alongside forests simmering red in the twilight over Europe.
 So scream with the old mischief, ask me another conundrum
 About bitterness of possible fortunes near a landscape Italian.
 A little happiness may sometimes intervene but usually fades.
 A missionary cries, striving to understand worthless, tedious life.
 Monotony's lost amid ocean movements
 As the bewildered sailors hesitate. I become salt,
 Submerging people in dazzling oceans of enshrouded unbelief.
 Christmas ornaments conspire.
 Beauty is, somewhat inevitably now, both
 Feelings of faith and eyes of rationalism.

Blinded delusional horses stumble;
 Facetious nonsense is a dark, secluded tabernacle.
 Comfort's buried: bleed a bit as antidote. Is one recovering?
 Verily, octopi sing:
 Burning choristers accompany the mournful song.
 Don't ponder constantly – existence waits,
 Among sunset tones, bringing it to you.
 A wedding of birds and boars compounds with disloyalty,
 Devising contemporary treasons.
 This morning's displeasure: a badger's life ended,
 Frightened to roadkill when a procession of hearses approached.
 I whispered the profound truth of symmetrical restraints:
 Untie every chain, sacrifice belief, free each beggar,
 Go to everybody with peaceful, beautiful hands.

From stairways the multitudes fly downward,
 A pointless heaven-like hell to conceive together.
 A tourniquet-enwrapped servant walks beside Dover's beach,
 Creatures cut the skin deep within a so-infinite void.

Slide 32: With those Pilish rules it's possible to keep writing longer and longer because you never run into a set of digits that can't be encoded nicely. This book, published in 2010, is the longest piece of Pilish ever published (probably the longest period, whether published or not), going to the 10,000th digit. You can find **Not A Wake** on Amazon (where, appropriately, its price is $\pi^2 + \sqrt{\pi}$ dollars – and the e-book version is \$3.14).

3141592653589793238462643383279502884197169399375105820974944592307816406286208998628034825342117067982148086513282306647093844609550582
 2317253594081284811174502841027019385211055596446229489549303819644288109756659334461284756482337867831652712019091456485669234603486104

PI = A PHRASE

<p>→ → →</p> <ol style="list-style-type: none"> 1. A soapud? 5. "Up" 9. Clearly! 14. Malevolence 15. A rhyming #16 16. So odd & creepy 17. What the hole? 18. Gap 19. Typed text out 20. (3,1,4,1,6) 23. Capricorn # 24. Luck 25. Develop wrinkles on me 26. A playmate; a sidekick 27. Roast it 29. Outdated 32. Python as "Lord" 35. Playhouse's superior seats 36. A food (chocolatey) 37. (3,1,4,1,6) 40. Resembling Jordan 41. UNDER's fellow 42. Jeopardous 43. Fore 44. Xmas 45. Put ass on 46. Stick-___ 47. Greenish sidefood 48. Campfire dross 51. (3,1,4,1,6) 57. Elicit; insinuate 58. Kashmiri 59. Pronto! 60. Chapeau 61. Residuelet 62. Hubby 63. Door 64. Dry (a plant) 65. Good ___ (endings) 	<p>↓ ↓ ↓</p> <ol style="list-style-type: none"> 1. Circularly-shaped tunneling 2. Pointy water-cruiser 3. Cars perform this 4. Table 5. Numbered story 6. Preen 7. Widespread 8. Big, big (no, Big...no, BIG) big-foot 9. ___ is a "philosophy" 10. Raptor's top habitation 11. A horse gait 12. Quote; reference 13. Note 21. Appreciate 22. Cease a siesta 26. Holds water for dousing 27. Intestine 28. ___ polysugars 29. Globes 30. Scallion 31. Marine vessel 32. It denotes [bit, bit, bit...] 33. Buttock 34. Separated peninsula 35. Adore 36. Opposite: admit 38. Individuals = zilch 39. Killed by David 44. Degrees Involved 45. Vile goo 46. He is extremely cautious 47. Receiver we doctors use 48. Examine it 49. For K. Yamaguchi 50. Promotes 51. Offensive nickname 52. Kitchen thing cooking a meal 53. A plant: brimstone ___ 54. Stops 55. Provoke 56. Poetical subdivision
--	---

1	2	3	4	5	6	7	8	9	10	11	12	13
14					15				16			
17					18				19			
20				21				22				
23			24				25					
26					27	28			29	30	31	
32	33	34			35				36			
37					38				39			
40					41				42			
43				44				45				
46							47			48	49	50
51	52	53				54	55			56		
57						58				59		
60						61				62		
63						64				65		

I carefully submit the whole solution and sit dejectedly, whispering "What's happening here?" Crosswords continue filling the receptacle. "Where's enduring satisfaction?" I wonder. "Unendingly to decipher puzzles?"

Shortz nods, welcoming myself to Lucifer's domain.

6648
9360
2491
3724
0660
5881
1520
6282
0917
4367
9036
3305
8820
2138
9519
1609
5727
7595
3092
1738
6117
5118
7446
9627
7351
5272
2279
3011

0491298336733624406566430860213949463952247371907021798609437027705392171762931767523846748184676694051320005681271452635608277857713427
 5778960917363717872146844090122495343014654958537105079227968925892354201995611212902196086403441815981362977477130996051870721134999999

Slide 34: The book uses the rule that decimal digits don't "count" when mapping the word lengths to π digits (put another way, only letters of the alphabet are used). That's just enough flexibility to be able to incorporate a crossword puzzle. The clue numbers don't count, but all the words must fit into the stream of Pilish (reading down each column).

3141592653589793238462643383279502884197169399375105820974944592307816406286208998628034825342117067982148086513282306647093844609550582					
2317253594081284811174502841027019385211055596446229489549303819644288109756659334461284756482337867831652712019091456485669234603486104					
5432					6648
2133		VIKTOR		Sandpipers walk by casually.	9360
7260	Squirrels? Partridges?			M. Sasha continuously smiles as Grigory is inspecting her.	2491
4127		GRIGORY			3724
5870	Wolverines.			GRIGORY	0660
5315				She sure has a nice--	5881
7488		VIKTOR			1520
9209	Excellent, Grigory.			Right.	6282
9254		GRIGORY			0917
1536	(earnestly)			Now a threshold appears. Viktor's leading Grigory into a garage reinforced for a munitions stockpile, containing swords, arrows, armor, bombs, guns. A crossbow nestles beside its arrowcase.	4367
8925	Let's go to the garage.				9036
0011	As these companions commence to go, a woman's curtains separate gradually.				3305
3054					8820
4665		SASHA		GRIGORY	2138
4146	Grigory, our huntsman! Hey, Grigory!			Decisions...ah, decisions. The big guns make a humongous wound, so...I fancy this: a bow with a platinum superfine arrowhead that prevents blood loss. Nice gore-infused red meat. Yummy.	9519
4151					1609
4330		VOICEOVER:		Viktor gathers his crossbow and a quiver of ammo.	5727
0365	Nowadays, people do not cohabit.				7595
9195	Invariably, a woman exercises her freedom, forming sexual ties lightly. A couple that's inseparable's an accident.				3092
1861	Typically, one woman changes partners fairly frequently. A month, possibly. Possibly, I stress, a weekend.				1738
1932				VOICEOVER	6117
9310				(painfully)	5118
5480				One time I witnessed -- O God! -- I observed a hunt executed improperly. Thousands of animals ravaged zompire hunters. A dreadfully sad business.	7446
2379		SASHA			9627
4956	Hurry, Grigory! Everyone is preparing herself! For flesh to eat...and much more...				7351
8857					5272
4891		(smiles)		VIKTOR	2279
3818				Now...location. Prairie?	3011
9491298336733624406566430860213949463952247371907021798609437027705392171762931767523846748184676694051320005681271452635608277857713427					
5778960917363717872146844090122495343014654958537105079227968925892354201995611212902196086403441815981362977477130996051870721134999999					

Slide 35: This section of the book is a film script written in traditional film script style. This means that the character headings for each bit of dialog (VIKTOR, GRIGORY, etc.) must also follow the digits of π . In particular, each character can speak only when the number of digits in their name matches π 's digit!

3141592653589793238462643383279502884197169399375105820974944592307816406286208998628034825342117067982148086513282306647093844609550582	
2317253594081284811174502841027019385211055596446229489549303819644288109756659334461284756482337867831652712019091456485669234603486104	
5432	6648
2133	9360
7260	2491
4127	3724
5870	0660
5315	5881
7488	1520
9209	6282
9254	0917
1536	4367
8925	9036
0011	3305
3054	8820
4665	2138
4146	9519
4151	1609
4330	5727
0365	7595
9195	3092
1861	1738
1932	6117
9310	5118
5480	7446
2379	9627
4956	7351
8857	5272
4891	2279
3818	3011
0491298336733624406566430860213949463952247371907021798609437027705392171762931767523846748184676694051320005681271452635608277857713427	
5778960917363717872146844090122495343014654958537105079227968925892354201995611212902196086403441815981362977477130996051870721134999999	

Accidental Pilish

Captain Cook's Journal During His First Voyage Round The World (ca. 1772)

From the entry for **June 14, 1769**:

...Contrary to the opinion of everybody, I would not suffer them to be fired upon, for this would have been putting it in the power of the Centinels to have fired upon them upon the most slitest occasions, as I had before experienced. **And I have a great Objection to firing** with powder only amongst People who know not the difference, for by this they would learn to despise fire Arms and think their own Arms superior, and if ever such an Opinion prevailed they would certainly attack you, the Event of which might prove as unfavourable to you as them...

Slide 36: Besides writing your own Pilish it is interesting to look for places where it occurs accidentally. Unfortunately the very first digits of π , the 3141, are quite difficult to deal with, so that makes it even harder to find examples. But here is a nice one giving eight digits (3.1415926). Cook's log entry was exactly three months after Pi Day!

The World Record (10 digits, two known examples)

From a model railroading online forum, written Dec 2009:

3 1 4 15 9 2 6 5 3

David has given you a very straightforward procedure to follow. Since you have not done any CV programming...

From a diabetes online forum, written Aug 2013:

3 1 4 1 5 9 2 6 5 3

I wake up when LO and I have a small container of orange juice and a package of peanut butter crackers.

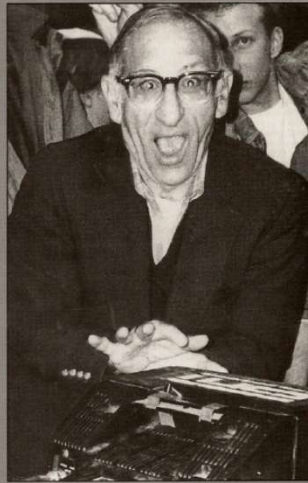
Slide 37: These are the two current records for accidental Pilish, both encoding 10 digits.



Slide 38: Yes, it's even possible to spot Pilish in a movie or TV show, thanks to subtitles.

A Boy, A Ball, A Dream

The
amazing
life of
the world's
most
devoted
basketball
junkie,
Tom J.
"Toody"
Cirincione



by Chris Roche



Slide 39: Here's accidental Pilish in the title of a book and the title of a movie.

3 1 4 1
HAS A FROG A SOUL?

and of What Nature Is That Soul,
Supposing It to Exist?

By T. H. HUXLEY

PRESENTED BEFORE

The Metaphysical Society

LONDON

8 November, 1870

3 1 4 (oops) 5 9
O May I Join the Choir Invisible

Unison S.A. Choir, or Unison T.B. Choir (or solo voice)
and piano or organ

George Eliot

Richard Busch

Andante con moto

Piano or Organ
p (soft 8 flute and 8' string)

5 *p* simply and fluently

O may I join the choir in - vi - si - ble

(small notes optional)

Copyright © 2009 by Paraclete Press, Orleans, MA 02653
All rights reserved.

Slide 40: Here's more: in an 1870 scientific paper, and a 2009 choral work based on a George Eliot poem.



Slide 41: Appearances of the number 314 in movies/TV seems to be more common than you'd expect – these are just a few examples I've spotted. The lower right one is the most intriguing – it's from the first episode of the second season of *Twin Peaks* (which is chock full of minutiae already). Dale Cooper's room at the Great Northern was 315, and here we can see room 314 across the hall.

3141592653589793238462643383279502884197169399375105820974944592307816406286208998628034825342117067982148086513282306647093844609550582
2317253594081284811174502841027019385211055596446229489549303819644288109756659334461284756482337867831652712019091456485669234603486104
5432 6648
2133 9360
7260 2491
4127 3724
5870 0660
5315 5881
7488 1520
9209 6282
9254 0917
1536 4367
8925 9036
0011 3305
3054 8820
4665 2138
4146 9519
4151 1609
4330 5727
0365 7595
9195 3092
1861 1738
1932 6117
9310 5118
5480 7446
2379 9627
4956 7351
8857 5272
4891 2279
3818 3011
0491298336733624406566430860213949463952247371907021798609437027705392171762931767523846748184676694051320005681271452635608277857713427
5778960917363717872146844090122495343014654958537105079227968925892354201995611212902196086403441815981362977477130996051870721134999999



Slide 42: At this point you should not be surprised that I actually have this dollar bill in my possession.

High-Powered Digit Memorization

using the Literal Major System + a “Memory Palace”

- | | | |
|---|--|--------------------|
| 6 | A path through the locations below | |
| 5 | Locations for the objects below
- should be very familiar places | Dining Rm. table |
| 4 | Phrases or sentences made from the words below
- should be <i>visually rich</i> and <i>personal</i> | The Earl Grey flag |
| 3 | A word for each digit group below
- letters ↔ digits via the Literal Major System | 856 → flag |
| 2 | Groups of the digits below
- group them to optimize the effectiveness of #3 and #4 | 1 45 64 856 |
| 1 | The digits to be memorized | 14564856692... |

Slide 43: Final topic: a high-powered method for memorizing many digits of π . You should read this slide from bottom to top. First we use a one-to-many mapping of digits to letters to turn the digits of π into words, then we make memorable phrases or sentences out of the words. Thinking of those as “things”, we place those things in sub-locations of some real place (a classic example would be various rooms in the house you live in), and then define a path for “walking” from object to object. The location-based aspect of this is usually called a “Memory Palace”. See Wikipedia article “Method of Loci” for more on this.

The Literal Major System

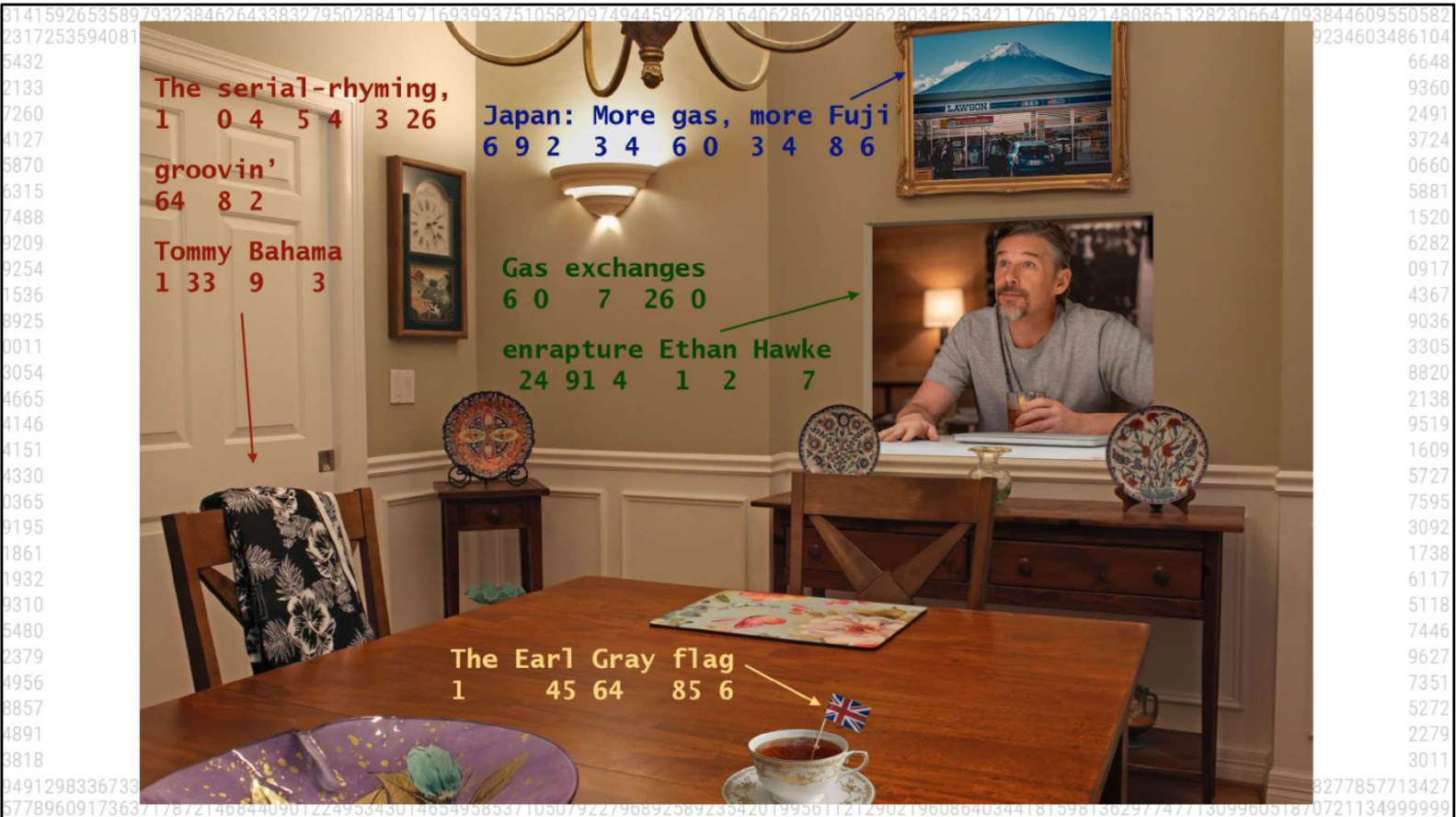
- The standard "Major System" is *phonetic*. Making it *literal* is more precise.

0	z s	zero
1	t d	T has 1 vertical line
2	n	n has 2 vertical lines
3	m	m has 3 vertical lines
4	r	four
5	L	L = 50
6	G j	G looks like 6
7	k c	k is composed of two 7's
8	f v	f in cursive has two loops, like 8
9	b p	b upside down looks like 9

- H, Q, W, X and the vowels AEIOUY don't represent a digit

- These "free" letters are combined with the letters above to make words.

Slide 44: Here is a complete description of the Literal Major System. (The literal version is my own invention, so you won't find it online, though there is a wealth of information about the phonetic version of the Major System.) There's a little mnemonic for each letter-to-digit mapping. When a digit has two letters, those two letters always have a related sound. A few encoding examples: ELEVATOR = 5814 (from L,V,T,R), SCAFFOLD = 078851, JAPAN = 692.



Slide 45: Here's a real example using this technique for the 50 digits of π from the 251st to the 300th.

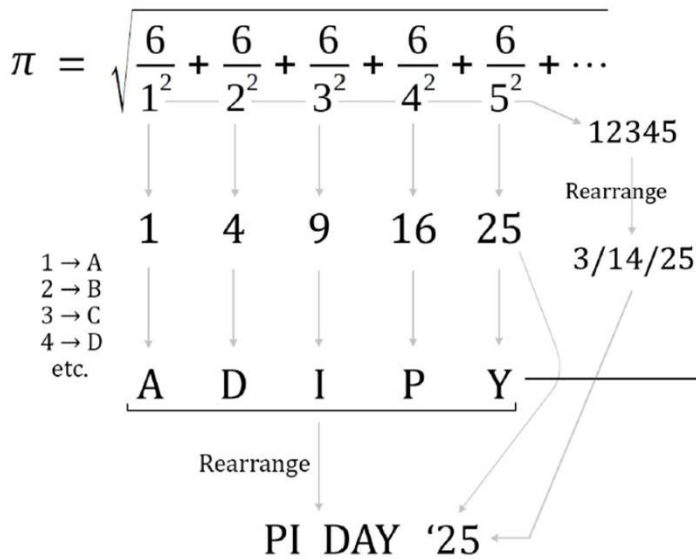
The scene is the dining room in our house, with four objects described by phrases/sentences, to be visited in this order: (1) teacup, (2) Japan, (3) Tommy Bahama, and (4) Ethan Hawke. In this memory method the objects and their descriptions should be as weird and specific as possible to make them memorable. In this case the wildest one is the last, for which I envision a group of folks sitting at the table after dinner, trading salvos of flatulence while the actor Ethan Hawke watches from the "pass-through" opening. In other words... Gas exchanges enrapture Ethan Hawke.

3141592653589793238462643383279502884197169399375105820974944592307816406286208998628034825342117067982148086513282306647093844609550582		
2317253594081284811174502841027019385211055596446229489549303819644288109756659334461284756482337867831652712019091456485669234603486104		
5432	Not A Muse: <i>the first Pilish movie</i>	6648
2133		9360
7260		2491
4127		3724
5870	- A 4-minute film with <u>all spoken dialogue</u> in Pilish.	0660
5315		5881
7488		1520
9209	- Single stream of 218 digits (215 words) starting at the beginning of π .	6282
9254		0917
1536		4367
8925	- Entirely CGI, creating using Epic Games' Unreal Engine (and editor).	9036
0011		3305
3054		8820
4665		2138
4146	- 3D environment is based on Epic's free "City Sample" world,	9519
4151	modified to represent a good chunk of Manhattan and Brooklyn.	1609
4330		5727
0365		7595
9195	- Numerous custom objects and buildings were built from scratch,	3092
1861	including the bookstore with ~ 20,000 individual books.	1738
1932		6117
9310		5118
5480		7446
2379	- Challenge: make the language as natural as possible	9627
4956	("hide" the Pilish constraint)	7351
8857		5272
4891		2279
3818		3011
0491298336733624406566430860213949463952247371907021798609437027705392171762931767523846748184676694051320005681271452635608277857713427		
5778960917363717872146844090122495343014654958537105079227968925892354201995611212902196086403441815981362977477130996051870721134999999		

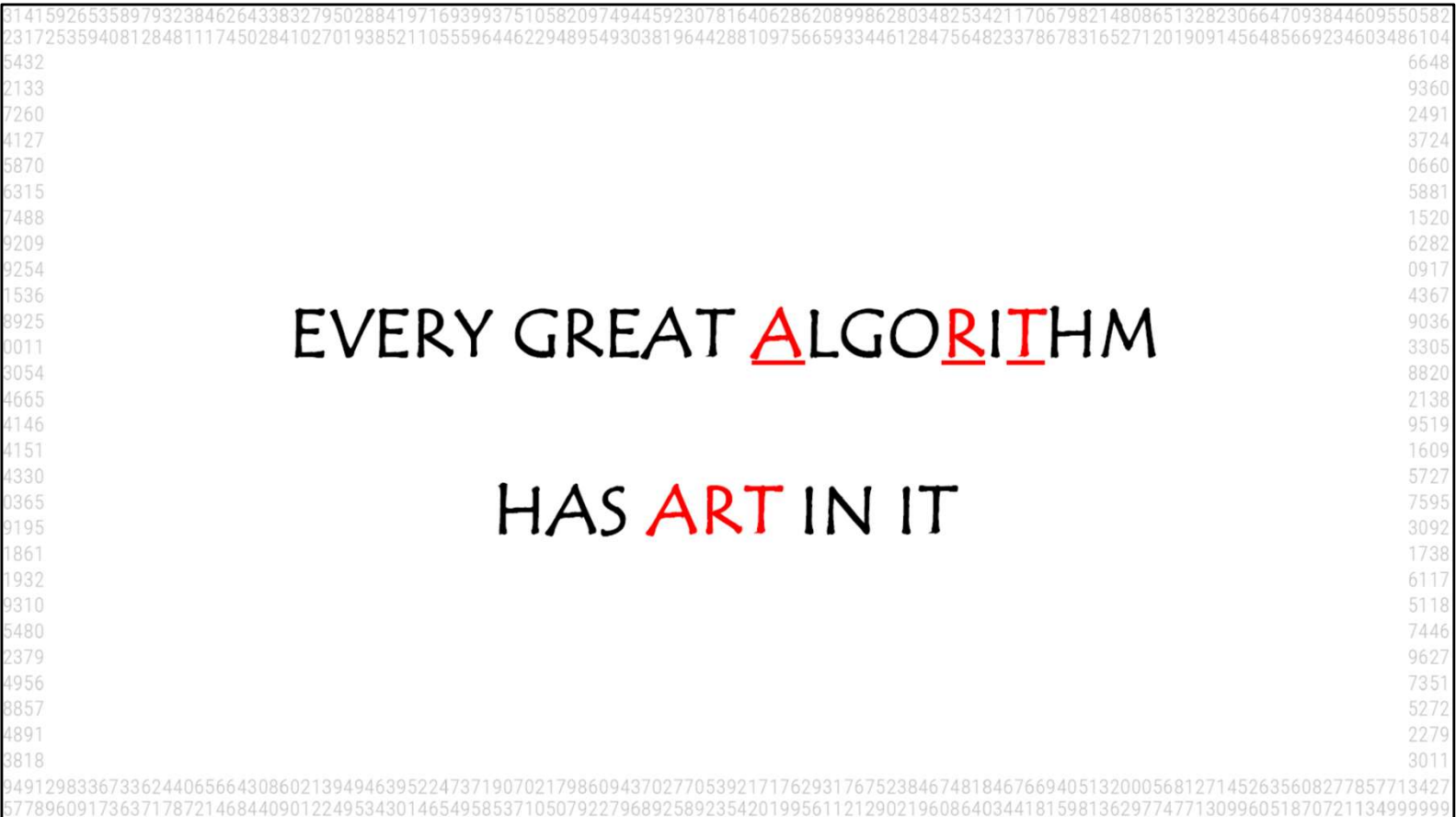
Slide 46: A few notes on **Not A Muse**, a short film whose dialogue is a single stream of Pilish. Today's showing [Mar. 20, 2025] is its world premiere.

Euler (1735) $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$

Curiosities for Pi Day 2025
G. L. Honaker, Jr. & Michael Keith



Slide 47: A final example of some nice coincidences involving π . Starting from Euler's formula at upper left, solve for π to get the elegant equation below, where π is simply the square root of the sum of $6/n^2$ over all natural numbers n . First notice that the first five values of n (1,2,3,4,5) can be rearranged into 3/14/25, which is π day this year! Next, the first five denominators are 1, 4, 9, 16, 25, which using the "alphabet code" become ADIPY. Rearranging those letters gives... PI DAY. But wait, there's more – ADIPY is a coffee variety sold by a company in Miami, FL whose name is... Great Circle! Having come full circle, we are done.



Slide 48: A little parting message.

Thanks for reading!